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ON MEAN RELATIVE AND ABSOLUTE PARALLAXES.

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In computing the mean parallax of a group of stars by comparing the radial velocities with the proper motions, it has been the custom to proceed in one of two ways. Knowing the apices of the sun's way, a great circle is passed through these points and the star. The total proper motion is then divided into two parts, one at right angles to the plane of this great circle, and the other in the direction of the circle. The former is called the tau component and the latter the upsilon component. The tau component is evidently free from any motion due to the motion of the sun, while the upsilon component contains all of the effect of the solar motion. Knowing the sun's velocity, the mean parallax of a group of stars distributed at random over the whole sky can be derived from a study of the mean algebraic upsilon component taken for each part of the sky. The formulæ used in this and the following method are found in "Stellar Motions," by W. W. Campbell, page 214 and following. It is seen that for the average of a group of stars $V_r = 4.74(\tau/\pi)$, where V_r denotes the radial velocity freed from the motion of the sun. For each star $V_m = 4.74(\mu/\pi)$, V_m being the total velocity across the line of sight. Let V_r be the total radial velocity, then for the mean of a group $V_m = 1.57V_r$. For, denoting the cross velocity freed from the motion of the sun by V_m, Campbell shows that in the mean, $V_m = 1.57V_r$, and $S_m = 1.57S_r$, S being the velocity of the sun with respect to any star, the subscripts denoting cross and radial motion as above. The individual values of S_r and V_r unite by addition and subtraction to form the values of V_r, and the quantities S_m and V_m unite in the same manner to form V_m. Hence we have $V_m = F(S_m, V_m) = 1.57F \times (S_r, V_r) = 1.57V_r$. The relationship deduced by Campbell, $V_m = 1.57V_r$, holds equally well if we choose a coördinate system fixed

with respect to the sun. In that case the two last-mentioned equations are identical; that is, the motion across the line of sight with respect to the sun is equal to the motion in the line of sight affected by the factor 1.57, for the mean of a large number of objects distributed and moving at random. The unit of V is kilometers per second, and of μ is seconds of arc per year. Substituting for V_m we find

$$\pi'' = \frac{4.74\mu''}{1.57 V_r} = 3.02 \frac{\mu''}{V_r}.$$

In case the data can be represented by curves somewhat similar to the probability curve, the average parallax is given by the formulæ of the last paragraph by comparing average radial velocities with average proper motions. If the data fit the probability curve well, we may use mean values of μ and V, as the ratio of the mean is the same as that of the average. We may still use mean values if, as is the case with the data under discussion, the curves for μ and V are of the same form and depart but little from the error curve. By "average" is meant such a value that there are as many data larger as smaller. The mean is 18 per cent. larger than the average, so we have for the mean parallax

$$\pi'' = 3.56 \frac{\mu''}{V_r}. \qquad (a)$$

Formulæ similar to the foregoing are only strictly valid in case the objects involved are distributed at random and moving at random. Any tendency toward systematic motion places certain restrictions on the use of these formulæ. For instance, if we had accurate proper motions of all the planetary nebulæ, the parallax as derived from the tau components would be too small, for the radial velocities of these objects show that they are moving sensibly parallel to the galactic plane. The apices of the sun's way lie near to this plane, therefore the tau components are very small in comparison with the radial velocities and total proper motions. The use of the upsilon components would be free from this latter difficulty. The use of total motions also avoids this difficulty, and has the further advantage of saving a great deal of labor.

By this last-named method, the average parallax was computed for each spectral class, using the data for all stars brighter than mag. 5.6. The "First Catalogue of Radial Velocities" by Voute was of great assistance in this work. It at once becomes evident from an examination of the data that the selection of the fainter stars for observation of the radial velocity has been largely influenced by the consideration of large proper motion. In the case of stars fainter than the sixth magnitude this effect is quite noticeable, and it begins to be seen in Classes *F* and *G*, even among stars of magnitude 5.2. The motions of the stars of large proper motion are directed more nearly across the line of sight than would result from random distribution, and so these data are not suited to studies like the present investigation. The effects of this type of selection are not of importance in the case of Classes other than *F* and *G* when stars brighter than 6.0 are considered; and the results for stars brighter than 5.6 of Classes *F* and *G* are not greatly influenced by selection.

To obtain the average velocity and proper motion, these data were plotted by spectral classes. The probability curve which best represented each group was found, and the average value was taken from the curve. The data in most cases fit the curve fairly well, the departure being in the sense of too many large velocities for the number of small ones. This excess may be due entirely to the element of selection among the stars fainter than 5.0. In the case of Class *G* the proper motions would suggest that we are dealing with two distinct groups of stars, the larger group being at about the distance of stars of Classes *K* and *M* of the same magnitude. The smaller group, about ten per cent. of the whole, appear to be much closer. Among stars of Class *F* there are just as many large proper motions as in Class *G* but there is no break in the curve. There are but few large velocities or motions among stars of Classes *K* and *Ma-c*. The data for the latter class fit the curves particularly well. The data for Class *Md* do not fit any smooth curve and they were not included in the discussion.

Taking the average values from the curves, the average parallax was computed as outlined in a previous paragraph. The results are given in column 2 of Table 1, and may be compared with the

results published by Campbell in *L. O. B.*, **6**, 132, 1911, and repeated in column 4 of this table. Columns 3 and 5 give the number of stars used to derive the values in columns 2 and 4, respectively. It is likely that the mean magnitude of the larger group of stars of the same class is the fainter, but the difference will be small. Campbell's results were derived from a comparison of the tau components with the radical velocities freed from the sun's motion. The values just determined were derived by comparing total observed radial velocity and total proper motion. Since the two methods give the same results as nearly as could be expected from so few data, no advantage in point of accuracy can be claimed for either. The use of total motions has the advantage of being less laborious.

TABLE I.

AVERAGE COMPUTED PARALLAXES.

Class.	Parallaxes from Total Motions.		Parallaxes from Tau Components (Campbell).	
	π.	No. Stars.	π.	No. Stars.
B–*B*5	0″.005	195	0″.006	312
*B*8–*A*	13	430	15	262
F	37	168	35	180
G	25	155	22	118
K	15	405	15	346
M	8	78	11	71

If now the observed relative parallaxes were at hand for all the stars in either of these groups, the difference between the mean observed and computed parallax would give the mean parallax of the comparison stars. While we do not have the parallaxes of all these stars, there are a sufficient number to permit the computation of a preliminary value of the correction which must be applied to reduce mean relative parallax to absolute. In the following discussion the data were confined to stars of magnitude 5.0 and brighter. Stars fainter than 5.0 have been selected for parallax observation almost entirely on the basis of proper motion, and stars of large total motion have been observed to the exclusion of others. But the most serious defect of the data for faint stars lies in the fact that the motions of the faint parallax stars are directed too nearly across the line of sight. The data which were used are free from this difficulty.

There are several observatories whose parallax determinations depend on comparison stars of about the same magnitude as those used at Allegheny. The mean value of the parallax found by these observatories was used in the following discussion. The excellent work of the Mt. Wilson Observatory was not used because the comparison stars in that series are much fainter. The stars for which the radial velocity, proper motion and parallax have all been observed were grouped according to spectral class. The mean radial velocity and proper motion was found for each class. This was not done by plotting curves as in the case of the stars discussed in the first part of the paper, but the simple arithmetic mean was used. Since the data are fairly well represented by probability curves, formula (*a*) may be used to derive the mean parallax. Table 2 gives the mean observed (relative) parallax by classes in the second column. The third column contains the mean values computed by formulæ (*a*), and column four shows the differences O — C. Column five shows the same differences on the basis of a correction of 0″.010 to be applied to the observed relative parallaxes in order to reduce to absolute. The results for Class *F*, indicating a larger correction are not entirely above suspicion and were not used in obtaining the mean correction.

TABLE 2.

OBSERVED AND COMPUTED MEAN PARALLAXES.

Class.	Observed Mean π.	Computed Mean π.	*O–C.*	*O–C* +0″.010.	Number of Stars.
*B–B*7.....	−0″.006	+0″.005	−0″.011	−0″.001	18
*B*8–*A*.....	+ 39	51	− 12	− 2	49
F.....	49	67	− 18	− 8	58
G.....	32	44	− 12	− 2	45
K.....	30	38	− 8	+ 2	85
M.....	+ 0.012	+ 0.021	− 0.009	+ 0.001	11

To many, a correction to reduce from relative to absolute parallaxes of the size of ten thousandths of a second will seem too large. But we cannot avoid the conclusion that a correction of this order must be applied to stars of Class *B*. Moreover, if the size of the correction were influenced by instrumental causes it would seem that Class *M* should indicate a correction considerably

smaller. This is not the case, yet the number of stars is too small to permit a definite conclusion. The larger correction is supported by the number of very small and negative parallaxes that have been found. This fact was checked by applying various corrections to the observed parallaxes and computing the cross velocities by means of the proper motions. Making the greatest permissible allowance for probable error in the parallax determinations and in the proper motions, and for the possible range of the distances of the comparison stars, the smallest value of the correction that would make the cross motions comparable with the radial velocities was found to be 0″.009.

About one half of the comparison stars used in determining the parallaxes discussed in this paper are of visual magnitude 9.7 or brighter on the Harvard scale. The number in each magnitude class was found by counting representative fields. Taking account of the number of stars in each magnitude class, the mean proper motion of the group was derived from the values given in Groningen Publications No. 30, page 99. Using this value, about 0″.030, and the most probable value of the mean radial velocity, 14.5 km., the mean parallax of the group becomes 0″.007.

It should be borne in mind that these data refer to the fields of the brighter stars, and that the fainter stars have been compared with fainter fields on the average. In a definitive treatment of the subject it would be necessary to take account of the magnitude of the comparison stars in each field in applying the correction to reduce from relative to absolute parallax. It will also be borne in mind that a larger number of data, insuring greater freedom from accidental and systematic error, may modify these results to some extent. Yet it is quite unlikely that a correction of less than 0″.007 will be found for the fields in which the comparison stars are all brighter than 9.5 visual.

It is perhaps not fully recognized that the value generally suggested for this correction to reduce relative to absolute parallaxes (*i.e.*, a correction of 0″.003 to 0″005) corresponds with a mean velocity of the comparison stars sixty per cent., or more, greater than that of the stars whose velocities we know. This value of the mean velocity results from the comparison of the mean proper mo-

tion, as given by Kapteyn, and the assumed parallax, 0″.003–5. There is no reason to suppose that the mean velocity of the faint stars is any greater than that of the brighter ones, although one might get that impression from an examination of the velocities which have been determined to date. This arises from the fact that the faint stars which have been observed for radial velocities have been chosen largely because of their large proper motion and they are mainly objects of great total velocity. Our direct knowledge of the velocity of the average faint star is practically nil.

The mean distance of the faint stars can be inferred from statistical discussions of the brighter stars, as has just been done, or computed more surely by means of radial velocities after representative objects have been observed. Yet there is no means of being sure of the exact parallax of the stars as faint as the ninth magnitude without actually comparing a considerable number of them with stars a good deal fainter, by trigonometric methods. The complete observation of all the so-called interesting objects on our programs will leave us about where we are now as far as the average faint star is concerned. On the other hand, an observatory devoting its parallax sessions to the subject could in a few years find the parallaxes of those objects of unusual interest, the representative star of each of the magnitude groups 6.5 to 9.5; that is, their parallax relative to stars of the 13th magnitude.

Conclusions.

A simple method of computing mean parallaxes from observed radial velocities and total proper motions is outlined, and results derived by this method are compared with those obtained by means of the tau component.

Observed and computed parallaxes of stars brighter than 5.1 visual are compared in order to find the mean parallax of the comparison stars. This value is seen to be 0″.010. Other considerations leading to a value of 0″.007 to 0″.009 are discussed.